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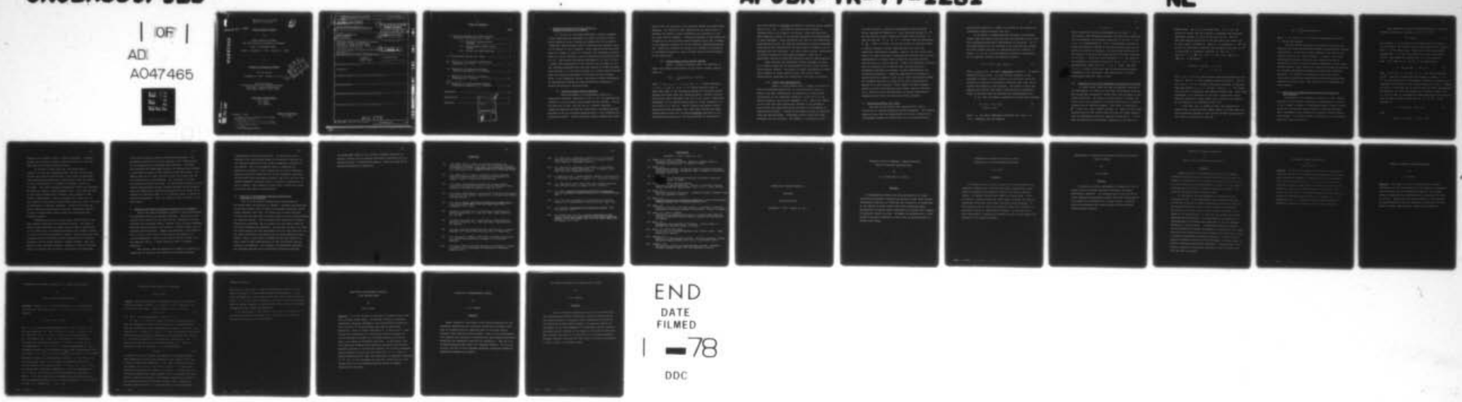

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CONTROL OF DYNAMICAL SYSTEMS

for the period

September 1, 1976 - August 31, 1977

Brown University
Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
Providence, Rhode Island 02912

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TABLE OF CONTENTS

I. Numerical Methods for Optimal Control of Systems with Delays (H.T. Banks).

In [1] and [2] the authors developed a general framework to treat approximation techniques for linear functional differential equation control problems and showed that the so-called "averaging" approximations could be discussed with complete rigor in the context of this framework. These approximations have been used for a number of years in a heuristic and sometimes incorrect manner by control engineers (see the engineering literature survey in [2]). The results obtained and discussed in [1] and [2] give a rather clear explanation of when and how one may expect to use the averaging approximations with some success in either optimization problems or in any other problems (e.g. system identification) where one requires computation of solutions of delay systems. Banks, in his subsequent efforts, has pursued further development of approximation methods for delay-differential equations in several directions as detailed below.

(i) Nonlinear system control problems.

Banks announced in [3] preliminary results on a theoretical framework (analogous to that in [1]) for approximation methods to treat nonlinear delay system control problems. During the past year he has, with the aid of a research assistant (partially supported by this Air Force grant), developed software packages to use the averaging approximations in this framework for nonlinear problems. Several nonlinear control examples have been

used to test the efficacy of the resulting method and computational packages. Our results to date indicate that for many classes of problems the scheme is quite satisfactory for nonlinear as well as linear control problems. A detailed treatment of the theoretical foundations of the nonlinear system framework along with a summary of our numerical findings are in the recently completed manuscript [4]. Banks will continue his efforts at a systematic development of alternative methods to treat nonlinear system control problems in the coming year.

(ii) Nonautonomous linear control systems.

Reber, a graduate assistant under the direction of Banks, has studied nonautonomous linear control systems (special cases are

$$\dot{x}(t) = \sum_{i=0}^{\nu} A_i(t)x(t-r_i) + B(t)u(t)$$

where $0 = r_0 < r_1 < \ldots < r_\nu$, A_i are $n \times n$ matrix functions, $i = 0,1,\ldots\nu$, and B is an $n \times m$ matrix function) such as those that arise in the engineering analysis of nonlinear control systems with delays when one linearizes about a nominal solution. Using factor space techniques (see [5]) he has given a theoretical development for an approach which offers a viable alternative to that given in [1] and [2]. Rather complete convergence results (along with error estimates, etc.) have been obtained for these methods which involve use of simple difference equations as the approximating equations for the delay systems (as contrasted to

the finite systems of ordinary differential equations used as approximants in [1], [2]). The immediate advantage with respect to implementation on the computer is obvious. Banks and Reber have begun numerical studies of the overall advantages and disadvantages of such techniques. Early indications are that for low dimensional approximating systems, the ordinary differential equation approximations may be more accurate when the same amount of computer time is employed with each technique. For higher order accuracy (necessitating higher dimensional approximating systems), there may be substantial savings in using the difference equation approximations.

Banks and Reber are developing software packages to carry out a careful comparison of these methods for both linear and nonlinear control systems. The results of these investigations should prove valuable to anyone having to deal with control and/or identification techniques for differential equations with delays.

(iii) Spline type approximations.

Banks, in joint efforts with F. Kappel (a visitor at Brown from February-August 1977) of the University of Graz in Austria, has been investigating the possibility of using other alternatives to the "averaging" scheme of [1] and [2] in a conceptual framework such as that proposed in [1]. During the summer of 1977 they made major theoretical advances on the problem of using spline type approximations as the basis for development of computational schemes. A theory which handles splines of arbitrary order has been developed. Convergence results along with order estimates are now available. For example, if splines of order k

are used to obtain a system of approximating ordinary differential equations of dimension N (with solutions denoted by x^N) for the original control system with delays (with solutions x), then one has that $x^N \to x$ as $N \to \infty$ and the order of convergence is $\frac{1}{N^k}$. Thus, in theory, one can use splines of arbitrary order to obtain extremely accurate approximations even with low-dimensional approximating systems. However, one must expect that there is a trade off between increasing difficulties with practical implementation of higher order splines and increased accuracy obtained via use of these splines. Initial considerations indicate that for numerical work piecewise cubic splines should be a reasonable compromise with respect to both implementation and convergence properties. Numerical experiments are underway to explore practical aspects of the use of these spline-based methods. The framework developed to handle these spline approximations appears to be quite general; Banks, Kappel and their students intend to pursue investigations using either types of functions (e.g., Walsh function approximations) to develop approximation schemes for both linear and nonlinear systems problems.

II. Bifurcation Theory (J.K. Hale).

Hale and his colleagues have continued their work on bifurcation in systems containing several parameters. The original papers by Chow, Hale and Mallet-Paret [6,7] were a stimulus for considerable research on bifurcation from an isolated solution

and have been applied to a number of problems in the buckling of rectangular plates and shells (see [8]).

In an effort to understand other problems in bifurcation theory, Hale and Taboas [9,10] and Hale and Rodrigues [11,12] have been considering bifurcation from families of solutions. Due to the complications that are involved in this more general problem, the first efforts have been to understand specific examples in detail. In [11,12], Hale and Rodrigues studied the bifurcation of $2\pi/\omega$-periodic solutions of Duffing's equation

$$\ddot{x} + x + p_1 x^3 + p_2 \dot{x} = p_3 \cos \omega t \tag{1}$$

where $p_1, p_2, p_3, |\omega-1|$ are small independent parameters. A complete analysis has been given. To carry out the analysis, it was necessary to obtain a priori bounds on the solutions, scale the variable to reduce the discussion to solutions near zero, and to exploit the symmetry in the equations. The manner in which the symmetry was used has some general implications in bifurcation theory as has been shown by Rodrigues and Vanderbauwhede [13].

In [9], Hale and Taboas considered the second order equation

$$\begin{aligned} &\ddot{x} + g(x) = \lambda \dot{x} + \mu f(t) \\ &f(t+2\pi) = f(t) \end{aligned} \tag{2}$$

where λ, μ are small independent parameters and $xg(x) > 0$, $x \neq 0$. Supposing that the equation

$$\ddot{x} + g(x) = 0 \tag{3}$$

had a 2π-periodic solution corresponding to an orbit Γ in $(x,\dot{x})$-space, they determined necessary and sufficient conditions for the existence of 2π-periodic solutions of (2) near Γ for (λ,μ) in a neighborhood of $(0,0)$. This problem is global in nature and cannot be reduced to finding solutions near a given point as in Duffing's equation. An interesting implication of the results is the fact that there may be 2π-periodic solutions of (2) near Γ which are nice functions of λ,μ but which do not approach a 2π-periodic solution of (3) as $(\lambda,\mu) \to (0,0)$ — a phenomena that cannot occur if $\lambda = k\mu$, k fixed. The case $\lambda = k\mu$, k fixed, is the usual type of one parameter problem considered in the literature. The implications in general bifurcation theory are given in [10].

III. Stability of Functional Differential Equations (J.K. Hale).

In recent years, there has been considerable interest in the development of a theory of functional differential equations for initial data belonging to spaces other than the space of continuous functions. The spaces of fading memory of Coleman and Mizel (which arise naturally in certain applications in material science) are an excellent example. This space is essentially $\mathbb{R}^n \times L[(0,\infty);g]$ with a weight function g. There are many other possibilities for the initial data and each such space requires that the fundamental theory be completely redeveloped. In [14], Hale and Kato have given abstract properties on the norm in a

Banach space $\mathscr{B}(-\infty,0)$ of functions from $(-\infty,0]$ into $\mathbb{R}^n$ which will ensure that the existence, uniqueness and continuous dependence results are valid for initial data in $\mathscr{B}(-\infty,0)$. They also discuss properties of the norm which will ensure that an orbit which is bounded in $\mathscr{B}(-\infty,0)$ is precompact in $\mathscr{B}(-\infty,0)$ as well as those which guarantee that asymptotic stability in $\mathbb{R}^n$ is equivalent to asymptotic stability in $\mathscr{B}(-\infty,0)$. If $\mathscr{B}_0(-\infty,0) = \{\phi \in \mathscr{B}(-\infty,0): \phi(0) = 0\}$ and $S(t): \mathscr{B}_0(-\infty,0) \to \mathscr{B}_0(-\infty,0)$ is defined by

$$\begin{aligned} S(t)\phi(\theta) &= \phi(t+\theta), \quad && t+\theta < 0 \\ &= 0 && t+\theta \geq 0 \end{aligned}$$

for $t \geq 0$, $\theta \in (-\infty,0]$, then the most important hypotheses on the norm in $\mathscr{B}(-\infty,0)$ is that constant functions belong to $\mathscr{B}(-\infty,0)$ and $S(t)$, $t \geq 0$, is a strongly continuous semigroup of operators such that there is a $t_0 > 0$ so that $||S(t_0)|| < 1$. The latter norm is easy to compute for the classical spaces used. Under these hypotheses, it is indicated in [14] that a qualitative theory should be possible in these spaces and it should be as complete as the classical one contained in Hale [15].

Paulo Lima [16], a student of Hale, has completed his dissertation on equations with fading memory, has shown that the Hopf bifurcation theorem is true, and he has made applications of this result to equations of the type

$$\dot{x}(t) = \int_{-\infty}^{-r} k(t-\theta) f(x(t+\theta))\,d\theta$$

where $r > 0$ and $k \geq 0$ is a function vanishing at zero and $-\infty$ and has one maximum.

Hale and Infante have begun an extensive program on the stability of linear differential difference equations with constant coefficients. Infante has already discovered the form for the Liapunov functions for linear systems. A student, Peter Tsen, has obtained necessary and sufficient conditions on the coefficients in order to ensure stability independent of the delays. Another student, Cerino Avellar is studying the stability question for singular perturbation problems in differential difference equations. The effects of variations in the delays can be significant in difference equations and differential-difference equations of neutral type. Our objective is to understand this basic problem.

IV. Stability of Feedback Structures (Control Generators) (J.P. LaSalle)

As outlined in our proposal of January 13, 1976, LaSalle returned to this problem (he has been thinking about it off and on for a number of years) of identifying stable ("good") control generators, and LaSalle and Max Palmer, a research assistant, spent considerable time attempting to develop a theory without much success. It, however, might be worthwhile to say something about what they tried to do.

They considered a control system described by a system of ordinary differential equations ($f\colon R^n \times R^k \to R^n$)

$$\dot{x} = f(x,u); \tag{1}$$

the n-vector x is the state of the system and the k-vector u is the control. The desired state of the system is the origin so that x is the state error. Assume that within some bounded neighborhood G of the origin there is an open loop control $u(t,x^0)$ that brings the system from x^0 to the origin in time $\tau(x^0)$; that is, if $\pi(t,x^0)$ is the solution of

$$\dot{x} = f(x,u(t,x^0)), \quad x(0) = x^0, \tag{2}$$

then $\pi(\tau(x^0),x^0) = 0$ for each $x^0 \in G$. The control function $u(t,x^0)$ need not be continuous with respect to either t or x^0, but the solutions are well-behaved (existence, uniqueness and continuity of solutions in the forward direction of time). Suppose now that it is possible (as it will be under the above assumptions) to synthesize the open loop control and to obtain a control law $u = \phi(x)$, so that in the absence of perturbations the two controls give the same solution curves; that is, if $\gamma(t,x^0)$ is the solution of

$$\dot{x} = f(x,\phi(x)), \quad x(0) = x^0 \tag{3}$$

then

$$\gamma(t,x^0) = \pi(t,x^0), \quad 0 \leq t \leq \tau(x^0).$$

These are the simplest types of control generators. Although neither of these may be "good" control generators, it was felt that this is the natural starting point.

As we know, in many cases the right-hand side of equation (3) will have discontinuities, and this raises mathematical and practical difficulties if the system is perturbed. Such discontinuous differential equations have been the subject of many investigations (in Germany, for example, after the V-1-bomb). The first successful mathematical theory was developed by André and Seibert, German mathematicians. Later work was done by the Russian mathematician Filippov, and by Hermes in this country, among others. LaSalle suggested that Palmer look at this theory of discontinuous differential equations, and at the work of Boltyanski on "regular" syntheses. Palmer did this and was able to make some improvements by giving a descriptive theory of a class of discontinuous vector fields and conjectured some stability results.

Although this was not a blind alley, LaSalle did not feel that anything essentially new was being learned. The class of vector fields covered by the theory and by Palmer's improvements is too restrictive, and something different, more closely related to reality of practical systems is needed. LaSalle concluded these questions were too difficult and too unchartered for a research assistant, and he shifted Palmer to another problem. Last July during a visit from Mark Aiserman, Institute of Control Sciences, Moscow, we learned that he and E.S. Pyatnitskii have developed

a new and more general theory of discontinuous systems. The conclusion we had come to in our research is the starting point of the investigation by Aiserman and Pyatnitskii. They have both published and unpublished results, and Aiserman has promised to send LaSalle copies of their papers as they are written. The answers here are not easy, and this is only an initial step for the investigation we had proposed. For the moment, at least, this research has been put aside. In our proposal of April 12, 1977 LaSalle suggested that another possible approach to this problem, where the mathematical difficulties should not be as great, is to replace the continuous model (1) by a discrete model (a system of difference equations). This he intends to take a look at in the near future.

V. Stability and Control of Discrete Processes (J.P. LaSalle).

LaSalle has made considerable progress in his investigations and expository writing on discrete processes. While many of the results can be said to be new, they are not surprising to anyone who knows the continuous theory. However, there are many instances where the discrete analog is not obvious, and some results are by necessity quite different. LaSalle has completed a first draft of Parts I and II of his book on "The Control and Stability of Discrete Processes". Part I is the linear theory and Part II is the nonlinear theory. A final rewrite of Part I is almost completed.

Mike Latina, under the direction of LaSalle, is working on completing the theory of the stability of discrete processes

(nonautonomous difference equations). At the moment, he is working on the relationship between the asymptotic stability of the limiting equations and the uniform asymptotic stability of the process. This is an essential step in completing the extension of Liapunov's direct method and in finally obtaining improved sufficient conditions for uniform asymptotic stability. The previous theory developed by LaSalle (see [17]) did not cover uniform asymptotic stability, which in practical applications is what is wanted. This research is going well, we know what needs to be done, and expect no difficulties.

VI. Stability of Nonautonomous Retarded Differential Equations (J.P. LaSalle).

This is a new area of research being investigated by Max Palmer under the direction of LaSalle. Although the definition of the skew-product flows is known for retarded functional differential equations (see [18]), the objectives of Sacker and Sell in doing this are such that the theory turns out to be unsatisfactory for stability theory. This is exactly what happened for ordinary differential equations, and so history will now repeat itself for functional differential equations. We know that the key is to find the right topology for the skew-product flow and that there will be the usual difficulties associated with infinite dimensional state spaces. Thus, at this point, although we know what needs to be done, there is some uncertainity as to the difficulties and the results to be expected. For instance, for difference equations, the limiting equations can be set-valued difference equations

(an unpublished result of Zvi Artstein, Weizmann Institute of Science, Israel) and for ordinary differential equations can be a generalized type of differential equation. What will they be for difference-differential equations?

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Hereditary Control Problems: Numerical Methods
Based on Averaging Approximations

by

H. T. Banks and J. A. Burns

Abstract

An approximation scheme involving approximation of linear functional differential equations by systems of high order ordinary differential equations is formulated and convergence is established in the context of known results from linear semigroup theory. Applications to optimal control problems are discussed and a summary of numerical results is given. The paper is concluded with a brief survey of previous literature on this class of approximations for systems with delays.

APPROXIMATION METHODS FOR OPTIMAL CONTROL PROBLEMS WITH DELAY-DIFFERENTIAL SYSTEMS

H. T. Banks

Abstract

We consider optimal control problems for systems described by functional differential equations and present approximation ideas based on elementary approximation results from linear semigroup theory. We discuss briefly three aspects of these ideas. First, we outline a general theory for linear system problems and indicate extensions to bilinear and certain nonlinear system problems. We next show how one can apply the resulting theoretical framework to particular classes of approximation schemes. We conclude with a brief discussion of examples and numerical results which support the efficacy of our proposed methods.

Approximation of Nonlinear Functional Differential Equation Control Systems

by

H. T. Banks

Abstract

We develop a general approximation framework for use in optimal control problems governed by nonlinear functional differential equations. Our approach entails only the use of linear semigroup approximation results while the nonlinearities are treated as perturbations of a linear system. Numerical results are presented for several simple nonlinear optimal control problem examples.

STABILITY IN NEUTRAL EQUATIONS

by

Jack K. Hale and Pedro Martinez-Amores

Abstract

Coupled systems of differential-difference and ordinary difference equations occur in various applications including the theory of transmission lines [1] and gas dynamics [2]. Stability of linear systems has been discussed by Brayton [1] using Laplace transform and the problem of absolute stability by Rasvan [12] using the frequency domain method of Popov.

In this paper, the same problems are discussed by the following method. By differentiating the difference equation, one obtains a system of neutral differential-difference equations. The desired solutions of the original problem are obtained by restricting the initial data to lie on certain manifolds in the space of all initial data. In this way, the methods for neutral equations may be exploited. A crucial step in the process is a transformation of variables used previously in [8] for studying stability in critical cases. In addition to obtaining results for the equations of Brayton and Razvan, we indicate how our method simplifies the discussion in [10] concerned with stability in general neutral differential difference equations. We also show how general difference equations can be considered as special cases of neutral differential-difference equations. Generalizations to arbitrary functional differential equations is also immediate when this approach is employed.

RESTRICTED GENERIC BIFURCATION

by

Jack K. Hale

Abstract. In the past few years, there has been considerable attention devoted to the existence of bifurcation for one parameter families of mappings. Concurrent with this development has been the extensive theory of the universal unfolding of mappings or generic bifurcation for families of mappings which depend on a sufficiently large number of parameters. The purpose of this paper is to discuss methods for determining the nature of bifurcation when the family of mappings has $k \geq 1$ parameters, but k is generally smaller than the number of parameters necessary to describe the universal unfolding.

GENERIC BIFURCATION WITH APPLICATIONS

by

J. K. Hale

Abstract: This paper is a set of lecture notes on generic bifurcation and its applications with the emphasis on equations involving more than one independent parameter. The general theory is discussed for problems which are degenerate to order one or two. Applications are given primarily to the buckling of plates and shells with the parameters representing external forces, loading, imperfections, curvature and dimension.

INTERACTION OF DAMPING AND FORCING IN A SECOND ORDER EQUATION

by

Jack K. Hale and Plácido Táboas

Synopsis. Suppose λ, μ are real parameters, f is a scalar function which is 2π-periodic, $xg(x) > 0$ for $x \neq 0$ and consider the equation

$$\ddot{x} + g(x) = -\lambda \dot{x} + \mu f(t). \tag{1}$$

For $\lambda = \mu = 0$, every solution has the form $x(t) = \phi(\omega(a)t + \alpha, a)$ for some constants a, α and $\phi(\theta+2\pi, a) = \phi(\theta, a)$. If there is an a_0 such that $\omega(a_0) = 1$ (i.e., there is a 2π-periodic orbit Γ in $(x,\dot{x})$-space) and $\omega'(a_0) \neq 0$, the problem is to characterize the number of 2π-periodic solutions of Equation (1) which lie in a neighborhood of Γ for (λ, μ) in a small neighborhood of $(0,0)$. A complete solution of this problem is given under the hypothesis that the function $h(\alpha) = \int_0^{2\pi} [\partial\phi(t,a_0)/\partial t] f(t-\alpha)\,dt \Big/ \int_0^{2\pi} [\partial\phi(t,a_0)/\partial t]^2 dt$ has a nonzero derivative except at a finite number of points α_j and $h''(\alpha_j) \neq 0$. The bifurcation curves in (λ,μ)-space are determined by the α_j and are tangent to the straight lines $\lambda = h(\alpha_j)\mu$ at $(\lambda,\mu) = (0,0)$. In general, the 2π-periodic solutions of (1) are not continuous at $(\lambda,\mu) = (0,0)$. The nature of this discontinuity is discussed in detail. It is also shown that a necessary and sufficient condition for a 2π-periodic solution $x(\lambda,\mu)$ to be continuous at $(\lambda,\mu) = (0,0)$ is that $\lambda/\mu \to$ constant as $\lambda \to 0$, $\mu \to 0$.

BIFURCATION NEAR FAMILIES OF SOLUTIONS

Jack K. Hale

Summary: Many investigations in bifurcation theory are concerned with the following problem. If $M(0,0) = 0$ and $\partial M(0,0)/\partial x$ has a nontrivial null space, find all solutions of the equation

$$M(x,\lambda) = 0 \tag{1.1}$$

for (x,λ) in a neighborhood of $(0,0) \varepsilon X \times \Lambda$.

If $\dim \Lambda = 1$; that is, there is only one parameter involved then the existence of more than one solution in a neighborhood of zero can be proved by making assumptions only about $\partial M(0,0)/\partial x$ and $\partial M(0,0)/\partial x \partial \lambda$. However, if $\dim \Lambda \geq 2$, then the problem is much more difficult and more detailed information is needed about the function M. A careful examination of the existing literature for $\dim \Lambda \geq 2$ reveals that the additional conditions imposed on M imply, in particular, that the solution $x = 0$ of the equation

$$M(x,0) = 0 \tag{1.2}$$

is isolated (see, for example, the papers on catastrophe theory). These hypotheses eliminate the possibility that Equation (1.2) has a family of solutions containing $x = 0$. Such a situation occurs, for example, for $M(x,\lambda) = Ax + N(x,\lambda)$, where A is linear with a nontrivial null space and $N(x,0) = 0$ for all x. There also are interesting applications where Equation (1.2) is nonlinear and there exists a family of solutions. For example, Equation (1.2) could be an autonomous ordinary differential equation with a nonconstant periodic orbit of period 2Π with the family of solutions being

Summary (continued)

obtained by a phase shift. When the differential equation in the latter situation is a Hamiltonian system, the parameters (λ_1, λ_2) could correspond to a small damping term and a small forcing term of period 2Π. To the author's knowledge, the first complete investigations of special problems of each of these latter types are contained in papers by Hale, Táboas and Rodrigues.

It is the purpose of this paper to begin the investigation of the abstract problem for Equation (1.1), especially to extend the results in the paper by Hale and Táboas.

PHASE SPACE FOR RETARDED EQUATIONS

WITH INFINITE DELAY

by

Jack K. Hale

Abstract: It is the purpose of this paper to examine initial data from a general Banach space. We develop a theory of existence, uniqueness, continuous dependence, and continuation by requiring that the space $\mathcal{B}$ only satisfies some general qualitative properties. Also, we impose conditions of $\mathcal{B}$ which will at least indicate the feasibility of a qualitative theory as general as the one presently available for retarded equations with finite delay in the space of continuous functions. In particular, this will imply that bounded orbits should be precompact and that the essential spectrum of the solution operator for a linear autonomous equation should be inside the unit circle for $t > 0$. Also, we impose conditions which imply the definitions of asymptotic stability in R^n and $\mathcal{B}$ are equivalent and that the ω-limit set of a precompact orbit for an autonomous equation should be compact, connected and invariant.

STABILITY OF NONAUTONOMOUS SYSTEMS

by

J. P. LaSalle

Abstract

Recent advances in the study of the limiting equations of nonautonomous systems and the invariance properties of positive limit sets of solutions motivate improving some of the known results connected with Liapunov's direct method. Here, we give improvements of a theorem due originally to Yoshizawa and some improved sufficient conditions for asymptotic stability and instability. What we do is to again generalize the notion of a Liapunov function. At the end, we point out how the new knowledge concerning invariance properties immediately sharpens our results.

NEW STABILITY RESULTS FOR NONAUTONOMOUS SYSTEMS

by

J.P. LaSalle

Abstract

The new invariance properties that have been established for nonautonomous ordinary differential equations greatly extend the range and power of Liapunov's direct method for the study of the stability of time-varying systems. An essential feature of the method is the establishment of a relationship between Liapunov functions and the location of the positive limit sets of solutions. The principal contribution of this paper is a theorem connecting Liapunov functions and positive limit sets of sufficient generality to close a gap in the present theory.